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540, 542

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Discussion of
"THE VIBRATION OF STEEL STACKS"

By Walter L. Dickey and Glenn B. Woodruff, Members, ASCE
(Proc. Sep. 540)

RICHARD JENNEY,[1] M. ASCE.—The authors have assembled in usable form about all that is known regarding the behavior of tall stacks in the wind, supported by observations of actual cases. The constructive suggestions for design and further investigation should be of considerable interest to the profession.

The incidence of serious vibrations is still, however, not entirely predictable; and the freedom from such troubles exhibited by some stacks is somewhat puzzling. In the belief that negative experience may also be of some interest, the writer would refer to a stack built near New London, Connecticut in 1953. This stack was very similar to the Moss Landing units, being 11'-5" top diameter by 244' in height; and having a flared base 80 feet high by 17' diameter at the bottom. There was a parabolic transition at the top of the flare. Plate thicknesses, from the top, were 74' of 3/8, 20' of 7/16, 20' of 1/2, 20' of 9/16, and 110 of 5/8" all laid up in 5' lifts. All material was Man-Ten steel. Ring stiffeners were applied internally at ten foot intervals. Butt-welded construction was used throughout. During construction, and until gunite was applied, the stack was guyed by three 3/4" steel wire ropes sloping about 40° and spaced at 120° on a guy ring 190' from the ground. After guniting with 2-1/2" of a Haydite-Lumnite mix, the guy ropes were slacked off to a twelve-foot sag and left in place for security until the behavior of the stack could be observed under a variety of conditions through at least one winter season. The foundation was an octagonal pad on 14" BP 73# piles 100 ft. long acting as friction units in nearly uniform, very fine, gray sand.

Although the design is obviously on the conservative side, by conventional standards, for a wind load of 25#/sq. ft. on the projected area, still the proportion is quite slender, and the calculated dynamic properties are such as might lead one to expect trouble. For unlined and lined conditions respectively these were:

Natural frequency	0.96 and 0.71 cycles per sec.
Period	1.04 and 1.41 sec.
Static deflection	8" and 6"
Critical wind velocity	39.4 and 29.1 mi. per hr.

Judging by these data, noticeable vibrations might have been expected from a 60 mile per hour wind on the completed stack, and preparations were made to take up slack in the guys if necessary to moderate the sway.

However, hurricane "Carol," which spread havoc in the New England area, resulted in no serious amplitudes of motion, at least so far as could be seen with the naked eye, and it was at no time deemed necessary to take up the guys.

The only features of this stack which might be considered at all unusual

1. Structural Engr., United Engrs. & Constructors, Inc., Philadelphia, Pa.

appear to be the rigid stiffening against ovaling, and between 1/16" and 1/8" excess plate thickness throughout. Of course, the platform at the top may have served to some slight extent to "spoil" the regular formation of vortices where they exert the greatest influence. Then, too, there may have been some appreciable energy absorption effected through the whipping of the guys, by the elastic working of the piles in the sand, and by the damping effect of the lining.

It seems unfortunate that proportions of stacks as dictated by functional requirements, together with the modern trend toward all-welded construction, both tend to produce undesirable elastic behavior within the critical range of response to winds of reasonably frequent occurrence.

Considering the extremely high cost of a plant outage occasioned by serious stack damage, substantial additional first cost may be justified to insure reliable and predictable behavior in a utility power stack. But, except in earthquake zones, such increased cost may easily result in steel stacks comparing quite unfavorably with corresponding concrete stacks, particularly for heights in excess of 200 ft.

The writer agrees with the authors that much further study is called for. It is suggested that, until further investigation has developed firm design criteria, it may be wiser not to use exclusively all-welded construction for slender steel stacks; except where special requirements dictate their use and the designer is made fully aware of the physical and economic hazards involved.

That this conclusion should follow so closely upon the completion of an apparently satisfactory stack may seen somewhat inconsistent; but it is to be considered whether any structure which can cause its designer continuing concern, and demand of its owner continuing vigilance, can be counted wholly successful. There persists a suspicion of unsoundness as to choice of materials, proportions and design assumptions, to the possible detriment of the over-all reliability of the plant.

JOHN N. PIROK,[2] M. ASCE.—Gas volume and draft requirements are the criteria that usually determined the proportions of stacks. Various rules of thumb were employed to adjust proportions to guard against aerodynamic difficulties. Designers cognizant of vibration induced by wind often found it difficult to get acceptance of needed changes in proportions to meet aerodynamic requirements. The theory presented gives basis to aerodynamic design. Dynamic stability will sometimes require increasing stack diameters beyond that actually needed to move calculated gas volumes.

The wind pressure formula engineers are most familiar with is:

$$P = KV^2 \qquad \text{(A)}$$

where P = wind pressure lbs. per sq. ft. of projected area

V = wind velocity m.p.h.

K = coefficient varying from 0.002 to 0.005

2. Structural Engr., Chicago Bridge & Iron Co., Chicago, Ill.

Equation (A) is similar in form to equation (1) used by the authors.

$$K = \frac{1}{2} \rho C_D \left(\frac{88}{60}\right)^2 \qquad (B)$$

If one uses $C_D = 0.35$ and $\rho = 0.00238$ as suggested by the authors coefficient K becomes 0.00086. The value suggested for C_D obviously is too low. C_D should be at least 0.60 when using equation (1) to calculate the wind pressure to be used for stack design.

Equation (1) should be corrected. The numerical coefficient shown in the authors' paper should be 0.00119.

The frequency of the alternating vortices as expressed by equation (3) must not be assumed to be precisely determined by this formula for any given stack. The wind velocity is not uniform over the full height of the stack.[3] The variation is given as

$$V_Z = V_{30} \sqrt[7]{\frac{Z}{30}} \qquad (C)$$

where V_{30} = wind velocity at 30' above ground

V_Z = wind velocity at Z feet above ground

For stacks 100' or less in height, the variation is not too significant. For stacks 200' high, the velocity at the top can be 30% greater than at ground elevation. For 500' high stacks the velocity at the top can be 50% greater than at ground elevation. The Strouhal Number S has not been established for all velocity ranges influencing stack designs. Frank[4] gives S = 0.15 for stacks. The authors used S = 0.19 in equation (3). Stacks are not always cylinders throughout their full height. The diameter varies from top to bottom in conical stacks. If resonance is to be avoided stacks should be designed for wind velocities at least 25% greater than shown by records for the area. A stack may be designed for a resonant velocity of 80 m.p.h., yet actual resonance may occur at 65 m.p.h. because vortex frequency cannot be precisely determined. If velocities of 65 m.p.h. frequently occur, the stack is certain to be in trouble. Wind velocity records must be studied for each location and natural permanent obstructions taken into account. Experience and judgment must still be reflected in the design.

The authors state ". . . . . most excessive stack vibrations have occurred in welded stacks. . . . . ." Is this all inclusive statement justified? Did excessive vibration occur because of welded construction? There is little published data on vibration of riveted stacks. There is not too much data on vibration of welded stacks. With welding came other changes that affect vibration characteristics of stacks. (a) Linings are made thinner and of lighter materials. (b) Steel plates in the stack are thinner. (c) Because linings are thinner, stack diameters are smaller. (d) More unlined stacks are built today than in era of riveted construction. (e) Better dispersion of ash and other products of combustion demands taller stacks. With increased demand

3. H.C.S. Thom, "Frequency of Maximum Wind Speeds" - Proc. Sep. 539.
4. Authors Paper

for taller stacks, vibration difficulties increased. The greatest change is due to lining reduction. The weight has been reduced over 50%. This weight reduction increases the natural frequency of stacks with resonance occurring at higher wind velocities.

The authors' equation (7) requires E and I to be expressed in foot units to be dimensionally correct.

Equation (8) is incorrectly printed. A summation sign was omitted in the numerator under the radical sign.

$$f = \frac{1}{2\pi}\sqrt{\frac{g\sum Wx}{\sum Wx^2}} \qquad (8)$$

The Rayleigh-Ritz method is a rapid procedure for determining the natural frequency of lined or unlined stacks. The calculated frequency of stacks lined with 2" of gunite is about 70% of the calculated natural frequency of unlined stacks. The usefulness of the authors' equation (8a) can be extended to lined stacks by determining the factor for materials other than gunite.

Equation (9) does not make allowance for stiffeners on stacks. Ovalling or breathing can be prevented with stiffeners. This is borne out by experience. A stack 100' tall and 4'0" in diameter vibrated severely as a cantilever. Measurements showed dilation or ovalling. Addition of stiffeners stopped both the ovalling and cantilever vibrations. There is a relationship between ovalling and cantilever vibrations.

The lining of stacks does more than provide for corrosion resistance. The lining backs up the plate increasing its ultimate strength from tubular buckling to the yield strength of steel plate. This may account for the fact there is no record of lined stack failures. The lining stiffens the stack against ovalling.

Lined stacks should be designed for lined conditions. They should be temporarily guyed during construction if there is danger the critical velocity could occur before the lining is installed. External damping devices are not good solutions. These are gadgets requiring maintenance and can be inoperative when needed most.

GEORGE S. VINCENT,[5] M. ASCE.—This paper represents a worth while advance in the knowledge concerning the action of wind on engineering structures. The specific data concerning the behavior of several steel stacks in winds of various velocities suggest certain design criteria which are pointed out by the authors. Further than this, they represent experimental data on a large scale extending the knowledge of wind action around a cylinder and shed further light on the oscillation of transmission lines and even on some forms of the more complex excitation of suspension bridge sections.

As shown by the authors, the Strouhal Number, S, is the most significant parameter relating the behavior of the stack in the wind. Some of the variation in the computed values of S arises from uncertainty as to the actual wind velocity at a given time and at the elevation at which the dominant wind effect occurs as well as the variability of the wind. In some cases the computed value differs so much from the standard range that explanation is sought. A

5. Bridge Engr., Physical Research Branch, Bureau of Public Roads, U. S. Dept. of Commerce, Washington 25, D. C.

closer study of the wind pattern and its resulting forces on a cylinder may be helpful to this end.

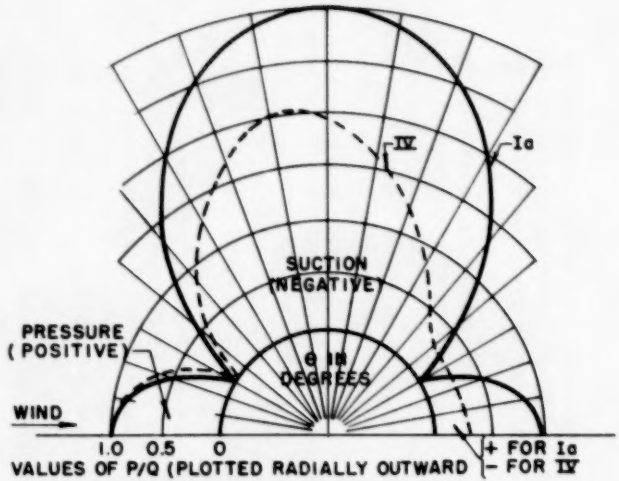
Sketch (a) of Figure 1 is taken from Pagon's more comprehensive diagram.[6] Sketches (b), (c), (d) and (e) show the stream flow around a cylinder at various stages. Sketch (b) shows the "potential flow" which exists briefly after the start of motion. This is the flow pattern which corresponds to Pagon's pressure diagram, I a, for ideal flow in sketch (a). The forces required to deflect the wind act toward the centers of curvature of the stream lines and the forces on the cylinder act in the opposite direction. At the center of the windward side the wind comes to rest at a "stagnation point" and the pressure therefore reaches its maximum positive value, $q = \rho V_2/2$. The same pressure pattern, occurring on the leeward side of the cylinder, accelerates the air at the leeward stagnation point from 0 to the stream velocity, V. It is in the negative pressure region at the sides that large values occur, up to three times the dynamic pressure. For the ideal or potential flow there is no net force tending to move the cylinder.

Due to viscosity the air flow near the cylinder is slowed down and air from the leeward stagnation area moves up stream (sketch (c)). The accumulating air in the leeward stagnation zone crowds the streamlines, reducing their curvature and modifying the pressure pattern as typified by Pagon's diagram IV. This accumulation must be released at intervals which naturally depend upon the wind velocity and dimensions of the area involved, giving rise to the periodic discharge of vortices as idealized in sketches (d) and (e).[7] The chance discharge of the first vortex from one side sets up an oscillation in the leeward stagnation zone which results in the subsequent alternation of discharges, one from each side per cycle of the Strouhal frequency. The pressure diagram IV in sketch (a) represents the average condition. Associated with the alternating vortex discharge is a variation in the pressure pattern alternating from side to side with respect to the longitudinal axis (parallel to the stream flow). The maximum value of this periodic pressure variation is expressed as an average transverse pressure in the authors' equation (4).

The authors' equation (6) using the magnification factor as π/δ is applicable only at or reasonably near resonance. The velocity range over which excitation occurs is broadened if the bending oscillation is large since the effect of the motion is to add the double amplitude to d in equation (3). For a uniform diameter d + 2A increases with the height as does also the wind velocity.

The reoccurrence of a mode at approximately twice the wind velocity indicated by the Strouhal number involves "sub-harmonic excitation"[8] in which the force is applied at twice (or some other multiple of) the frequency of the oscillation. Such excitation is possible only to the extent that the exciting force or the oscillation or both may be non-linear. Thus, although the wind forces would be increased four fold as noted by the authors in the last paragraph on the Contra Costa stacks, their net effectiveness in exciting motion might be quite small. The relative ineffectiveness of sub-harmonic excitation

6. "What Aerodynamics Can Teach the Civil Engineer" - W. Watters Pagon. Engineering News-Records, March 15, 1934, Figure 3.
7. Photographs taken with the fluid and camera stationary and the cylinder moving show a strong circular pattern affecting all of the stream lines but the pattern is elongated when considered with the fluid in motion and the cylinder stationary.
8. "Mechanical Vibrations," J. P. Den Hartog, McGraw Hill Book Co.

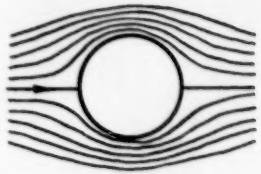


Ia - POTENTIAL FLOW $P/Q = 1 - 4 \sin^2\theta$, $C_d = 0$

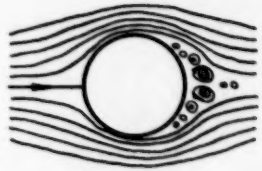
IV - PRANDTL & BETZ; GASHOLDER MODEL, HOLE NO. 2, 11.8' DIA.

98 FT/SEC. ($R = 605{,}000$) $\frac{\text{LGTH.}}{\text{DIAM.}} = 3.8$

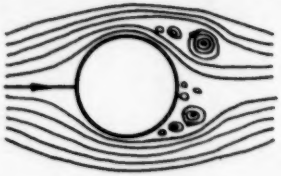
(a) POSITIVE AND NEGATIVE PRESSURE ON A CYLINDER



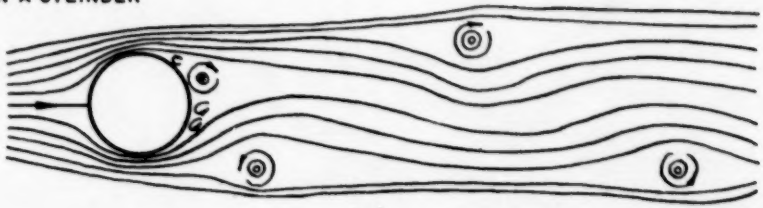
(b)



(c)



(d)



(e)

Fig. 1

undoubtedly is the reason for the low magnification factor noted by the authors in the behavior of the 9 ft. stack on Guam at a velocity of about 120 mph. (Incidentally, it would be of interest to know the subsequent history of this stack. If it has weathered several typhoons fatigue may become an important factor.)

Frank's advice that the frequency of the stack be kept as low as possible seems intended to permit resonant excitation at low velocities having little power with the expectation that subharmonic excitation at higher velocities will be inefficient.

The pressure pulsations which cause breathing or ovaling occur twice per vortex cycle; i.e., once for each vortex. If this is taken into account the data for breathing of the Moss Landing stacks would indicate Strouhal numbers of 0.225 and 0.175 for velocities of 37 fps and 48 fps, respectively. The breathing of the Guam stack was evidently sub-harmonically excited. If, for example, the effective velocity was 108 mph and S is assumed to be 0.2 then the vortex cycle frequency is found to be 210 cpm. If the maximum estimated diameter of the oval, 11 ft., is assumed to be the effective diameter the frequency of 210 cpm would hold up to a velocity of $\frac{11}{9} \times 108 = 132$ mph. It seems evident that two complete vortex cycles (4 individual vortex discharges) occurred per cycle of breathing.

The behavior of stacks in groups brings to mind the oscillation of individual strands of the open stranded cables of the Peace and Liard River suspension bridges on the Alaska Highway.[9] The cable of the former consists of 24 - 1 7/8" strands arranged in six horizontal layers of four strands each spaced 3 1/2" centers both ways. The Liard Bridge cable has 12 - 2 1/16" strands arranged in 4 horizontal layers at the same spacing. Before spacer blocks were placed at the middle of each panel (about 40' long) the individual strands vibrated frequently in the wind. Strands in the windward tier of the cable were never seen to vibrate but those in the adjacent tiers did, evidently under the influence of the vortex trail of the windward strands. At Peace River practically all of the individual wire breaks were nearly equally divided between the two interior tiers while at Liard River over half were in the middle tier. Apparently the tier next to the windward one was most affected. Action was greater and more frequent in the windward cable; evidently turbulence shed by this cable as a whole weakened or destroyed excitation on the leeward cable.

Since both the clear length between hangers and the tension in the strand increases toward the tower the frequency was fairly constant (588 cpm measured at Peace River and 420 cpm at Liard River) and motion was observed in practically all panels, depending upon the direction of the wind. Usually only half a dozen panels were involved at a time. Since the vibration was noticed in winds from a few miles per hour up to 30 or 40 mph it is evident that the excitation must have been subharmonic.

Tests on bridges have shown distinctly greater damping on pile foundations than on piers founded on rock and a greater effect of foundation conditions is to be expected when the foundation directly supports a vibrating cantilever.

The increase in damping obtained by adding a 1.92" gunite encasement to a 6" steel pipe (authors' Table I) must be much greater than can be expected to result from guniting a steel stack since the thickness of gunite used on the model was 32% of the outside diameter of the steel pipe, although the relative thickness of steel and gunite was not so much out of line.

9. "Cable Vibrations on Alaska Highway Suspension Bridges" Raymond Archibald, Engineering News Record, September 2, 1948.

R. H. SHERLOCK,[10] M. ASCE.—The subject matter of this paper is still in the pioneer stage. It will probably continue so for some years to come. Nevertheless engineers must proceed with the design and construction of high stacks since height is one of the considerations involved in the proper dispersion of stack gases. Under these conditions it is especially important that engineers in this field make available to the profession the results of their experience and their thinking. The authors have provided a welcome addition to the meager literature in this field.

The writer wishes to discuss Item 4 under the authors' heading of "Further Investigations." It is the general practice for writers on this subject to accept the theory of the Karman Vortex Trail as the explanation for the exciting forces necessary to produce vibration across the windstream. The theory has been well validated for values of Reynolds Number below the critical value. It can be said, also, that even above the critical value, the cylindrical smoke stacks vibrate as they would be expected to do if the flow around and behind the stacks followed the Karman Theory. In the present state of the art and its underlying sciences the Karman Theory serves as a useful framework on which to hang coefficients based on experience. It cannot be said however that at these high values of Reynolds Number, the reality of the Karman Trail has been established.

A cylindrical stack constitutes an obstruction in the air stream. The air is forced to flow around it and tends to resume the undisturbed conditions of flow as soon as possible downstream from the obstruction. If the air were a perfect gas, there would be no loss due to viscosity and friction in the boundary layer, and the air would resume its undisturbed condition of flow without separation of the boundary layer from the surface of the stack. In such a case there would be an increase in the velocity and kinetic energy of the layers adjacent to the sides of the cylinder on the upstream side, with a corresponding decrease in static pressure, so that at the cross-wind axis of the cylinder these layers would have gained the exact amount of kinetic energy necessary to permit them to return to their original line of flow against the increasing static pressure at the downstream surface.

Actually, however, the air is not a perfect gas but is viscous, and the conditions of flow are different. The boundary layers lose energy through friction and viscosity on the upstream sides and are eventually forced to separate from the surface of the cylinder, thus enclosing a region in which the static pressure is lower than that of the freely flowing air but is higher than that of the fast moving enclosing layers.

Due to its momentum, each layer is able to support a difference in pressure between the air spaces inside and outside the layer. However, this difference in pressure cannot be supported indefinitely and, in those cases where the Karman Theory holds true, the layers turn inward and roll up into vortices with their axes of rotation parallel to the axis of the cylinder. The vortices flow downstream and eventually disintegrate as the velocities in the component parts of the wake approach the velocities in the undisturbed airstream. This formation has come to be known as the "Karman Trail" and the reality of its existance at Reynolds Numbers below the critical value has been established under laboratory conditions a number of times.

At the Reynolds Numbers which exist in the case of smoke stacks during high winds the trailing vortices are occasionally outlined by the entrapment of smoke within them. They appear as vertical tails spinning their way down

10. Prof. of Civ. Eng., Univ. of Michigan, Ann Arbor, Mich.

wind. They rarely show even a quasi periodicity even in the case of rough stacks. At one plant where there are eight vertical angles on the outside of the stack, which gives the effect of great roughness, these trailing vortices can be seen frequently. However even if the reality of the vortex trail were established at high Reynolds Numbers in a wind tunnel, there might still be considerable doubt as to its reality in the free atmosphere during high winds.

There are a number of differences between the conditions of a stack in the free atmosphere and of a cylinder in a wind tunnel:

1. The stacks are not stationary but are vibrating, often through a considerable amplitude.
2. The stacks are not always cylindrical but sometimes have the shape of truncated cones so that the vortices would be generated at different frequencies along the height of the stack, according to the Karman Theory.
3. During wind storms there is usually a strong temperature lapse rate in the atmosphere with consequent turbulence, which varies throughout the height of the stack.
4. The wind is gusty so that the velocity is constantly changing.

It does not follow that there are no lateral exciting forces even though the Karman vortices do not have an opportunity to form in the immediate wake of the stack. Other hypotheses are possible. For example, if it is assumed that the Karman vortices do not form downwind from the stack, there still would be the certainty that the boundary layer would form on the upstream side and on the lateral sides of the stack and that these layers would be forced to separate from the downwind surfaces of the stack. They would flow downwind as layers of high momentum capable of enclosing a low pressure zone for some distance. The presence of this low pressure zone is frequently marked by a bulbous protrusion on the bottom of a horizontal smoke plume close to the stack. As the stack vibrates across the wind the angle of incidence of the wind will change periodically and the period will be controlled by that of the stack. Likewise the points of separation of the boundary layers will move back and forth with the period of the stack. This should cause a difference in the simultaneous velocities of the boundary layers on the two sides, with a consequent periodic difference in pressures. In this concept of the exciting mechanism the period of the alternating pressures would always be controlled by the period of the stack. The incidence of catastrophic vibration would not depend upon resonance between the period of the vortices and that of the stack, but upon the frequency of occurrence of atmospheric conditions favorable to this orderly behavior of the boundary layer. The role of damping would be the same under both hypotheses.

Under their heading "Further Investigations" the authors might well add to their very excellent list such questions as these:

To what extent does the turbulence usually associated with high wind velocities inhibit the development of large periodic lateral forces?

What is the probable frequency of occurrence of those types of high winds which are favorable to the development of large periodic lateral forces?

The first question deals mostly with aerodynamics, the second one with meteorology.

W. WATTERS PAGON,[11] M. ASCE.—Because of the interrelation between Separates 540 and 541 the writer will discuss only the former as being the more general, but with reference from time to time to the latter. The pair are a welcome addition to the recorded observed facts for cases of stack swaying or ovaling, and the integration of these facts with an attempted theory, whereby the facts may be brought within the purview of the stack designer.

The design problem is one of many variables, which seldom have even little similarity, between one power plant and another. A few of these are: the probable velocity of the wind and its turbulent or non-turbulent nature; the orientation of the stack group to probable winds of suitable velocity; the height/diameter ratio; the ratio of bell length to cylindrical length; the soil characteristics (for ground level foundations) or the vibration frequency of roof supports (as in Ref. 2 of 540); the shell thickness; the presence or abscence of ring stiffening to prevent ovaling; the effect of a lining on period and damping and rigidity; the effect of welding versus riveting; etc.

To the excellent recording of the facts of these cases quoted, the writer feels it to be of value to add a few remarks by way of amplification of the factual record, and also to call attention to much theoretical basis that is still unused for the creating of a theory that can be used with confidence in his results by the designer.

(a) Comments in both papers refer to the damping in riveting joints, and attention should be called to the fact that in Ref. 2, the "Baltimore Stacks," the stacks were lap riveted; and also that the swaying was apparently due entirely to the flexibility of the roof structure, and was completely corrected by roof stiffening, so that no further trouble was encountered; also that ring reinforcing was added to correct ovaling. As these stacks approached their final age of about fourteen years, and when rusting in the vicinity of the transverse lap joints became increasingly serious (since they were unlined), still more rings were added from time to time, which somewhat increased the moment of the stack weight, but had no effect on swaying.

(b) The most serious deficiency in the basis for a theory is the entire lack of test data on the influence of one stack on its neighbors. Does four diameter spacing (scaled from Separate 541, Fig. 6) for Moss Landing, or 4.45 diameter spacing for Baltimore stacks, have significance? The swaying of the latter resulted when the eddies on one side from the windward stack in a quartering (i.e. 45^{o}) wind enveloped the leeward one; and ovaling was mostly confined to the leeward one (being reversed for wind in the reverse direction). The smoke trails which were visible indicated that the stack spacing was about equal to the eddy spacing.

To generalize, there is great deficiency of knowledge as to what happens when there are two, or three or more stacks in a row. We have no satisfactory test data for multiple cylinders, either in a row in the line of wind, or in "echelon" for a quartering wind. Later than the publication of these two Separates there has just been published in Journal of the Aeronautical Sciences, Feb. 1955, Vol. 22, No. 2, pp. 124-132, a summarization "On the Wake and Drag of Bluff Bodies," by Anatol Roshko, which is an amplification of previous data published in NACA TN 3168, "A New Hodograph for Free Streamline Theory"; in TN 3169, "On the Drag and Shedding Frequency of Bluff Cylinders"; and TN 2913, "On the Development of Turbulent Wakes from Vortex Streets," each by the same writer. The writer will outline briefly what he has derived from these studies, and will call attention also to the fact that

11. Cons. Engr., Baltimore, Md.

they are based upon the same method as was used in his paper, Engineering News Record, Dec. 27, 1934, "Wind Tunnel Studies reveal Pressure Distribution in Buildings," Fig. 3, and also a flat plate (not published) which yielded the same results as now found by Roshko in TN 3168, Fig. 1, "Pressure on Flat Plate."

Briefly, Roshko has reverted to the studies of Kirchhoff and Helmholtz on the drag of a flat plate, who assumed that the "base pressure" on the leeward face of the plate is the same as the pressure "at infinity" in the free stream. However, Roshko changes this assumption to one wherein this base pressure is some function of the velocity within the wake in the vicinity of the body, and before the vortex street has degenerated into mixed turbulence from interaction of the two sheets of alternating eddies. He recognizes fully that his results apply only in the vicinity of the body, but it is here that the base pressure is developed, and there results in Fig. 1 of TN 3168 the same agreement that the writer found when plotted to the results of Fage & Johansen, British R. & M. 1143, "The Structure of Vortex Sheets." In Journ. Ae. Sc., Vol. 22, No. 2, Figs. 2 and 3, page 127, Roshko plots the coefficient of drag, C_D, and the width of the wake for a "normal" (i.e. 90°) plate, for a 90° wedge, and for a circular cylinder.

With this foreword the writer turns now to Roshko's discussion of the Strouhal Number, S. In Fig. 4 Roshko shows the coefficient of drag (at Reynolds Number 14,500) for a circular cylinder, to whose downwind meridian is attached a "splitter plate" extending aft for a length of five diameters (5d) of the cylinder, in comparison with the same cylinder without the splitter plate. At a distance of about 1.2d downwind he finds the center line wake pressure, C_p, to be -1.2 without the splitter plate, as against -0.45 with it.[12] In his Fig. 5 he shows additional results for a splitter plate of length 1.14d, that is variously located with the distance, c, measured from the lee of the cylinder to the lee end of the plate, varying from 2.14 to 7.0d. The results are extremely interesting.

First: he plots the wake pressure coefficient, C_{PS}, varying from -0.72 at $c/d = 1.7$, the shortest diatance, to -0.52 at 3.8d; but there then is a sudden abrupt change to -1.0 at this same distance; and then a slow rise to -0.93 at $c/d = 7.0$

Second: He plots the Strouhal Number, S, at $c/d = 1.7$ as 0.17; then 0.12 at $c/d = 3.8$; then again a sudden rise here to 0.19; and finally 0.21 at $c/d = 7.0$.

These results are cited here as indicative of what the writer feels to be of the utmost importance; namely, our complete lack of knowledge of what values should be considered for the eddy frequency, and the drag coefficient (or base pressure) of a cylinder when there is interference to the formation of the Karman vortex street. The writer trusts also that means can be found for extending these results to give consideration to the results of an "oblique" or quartering wind on two cylinders; and then to what is the effect on one placed in the lee, and then three or four in the lee.

A quotation is appropriate here from Journ. Ae. Sc., Vol. 22, No. 2, page 125, in which Eq. 1 is $C_{PS} = 1 - k^2$, where k is the ratio of U_S, the velocity of free streamline at separation, to the velocity "at infinity"; and also d = cylinder diameter, d' = "distance between free streamlines"; also S is the Strouhal Number, and S* is the wake Strouhal Number, formed from n frequency, d or d' dimension, divided by U at infinity and U_S. The test results deal only with a Reynolds Number less than 20,000, and the study for circular

12. The numerical values which the writer will use have been scaled from this Figure.

cylinders is not completed if separation occurs beyond the 90° meridian on the cylinder. Quoting: "* * * the same treatment should apply to higher Reynolds Numbers. * * * the intersection between wake and cylinder should be qualitatively the same as at lower Reynolds Numbers. For instance, some recent measurements (Delaney & Sorenson, "Low Speed Drag of Cylinders of various Shapes," NACA TN 3038, Nov. 1953) on circular cylinders above the critical Reynolds Number show that the Strouhal Number is about 0.4 when the drag coefficients is 0.4. The typical value of the base pressure at these Reynolds Numbers is -0.4 from which k = 1.2. Assuming that the universal Strouhal Number, S* = 0.16, applies here, the wake width may be calculated from Eq. 4. It is found to be $d'/d = 0.6$, which is reasonable, and supports the view that the processes in the wake are essentially unchanged at Reynolds Numbers above critical." For explanation, $S^* = (S/k)(d'/d) = nd'/U_S$ with n the vortex shedding frequency.

One other element that is uncertain is the shape of wake and the shedding frequency behind the central cylinder (of a set of three, etc.), in comparison with that of the two outer cylinders, since the former wake is compressed between the others. If the several stacks are found to be subject to wakes that vary markedly, so that swaying is not in phase for all of the group, then the excellent longitudinal bracing shown in Separate 541, Fig. 6 could be amplified so as to secure a horizontal beam of width equal to the stack diameter, and designed for the condition when adjacent stacks are in reverse phase.

The writer is aware that this seemingly is very verbose, but he hopes that it may serve to indicate still further research required in addition to what is suggested in "Further Investigations" (540-15). He suggests that definite steps be taken to amplify the work which has been carried on by Roshko in the references cited, and thereby to formulate anew a theoretical basis for determining the drag on flat plates, perforated plates (i.e. trusses), cylinders, and other "bluff bodies," that will bring the Kirchhoff-Helmholtz theory up to date; and hopes that the structural engineer may be afforded the same kind of benefits that have been accorded to the aerodynamic engineer.

The two papers cited are well documented and contain material of value to other designers; the owner of the stations is to be complimented for furnishing means and opportunity to experiment and formulate the results; and there is still need for much further information before a theory can be fully developed.

One source of information can be added to the foregoing for further amplification; namely a set of tests on two parallel cylinders exposed to a wind normal to the line joining them, and for velocities of 20 to 140 fps, in Journal of The Aeronautical Sciences, June, 1946, pp. 289 to 301, "Vortex Frequency in the Wake of two parallel Cylinders at varied Spacing normal to an Air Stream," Herman M. Spivak.

The cylinders were 1 3/8" commercial brass tubing, 4.5 ft. long, i.e., for the full height of the wind tunnel, or for "infinite" length of cylinder. There is given a discussion of the Strouhal Number, S = fd/U, or frequency, diameter and free stream velocity, and "gap spacing," G, divided by the diameter, or r = G/d, and for Reynolds Number ranging from 15,000 to 93,000 with constant results. No information was obtained for higher values, but the above discussion deals with numbers up to about $R = 2 \times 10^5$.

There is an abrupt discontinuity in S at r = 0.909, indicating that for a gap spacing of about one diameter and over the two wakes have little effect on each other. In twelve diagrams are plotted the frequencies observed at various points in the wake or wakes, and although there are many scattered instances of "doubled" frequencies, and some instances of unrelated values,

there are tabulated in Table 2, page 293, the predominant values, as follows:

r =	0.000	0.020	0.045	0.068	0.091	0.182	0.273	0.364	0.455	0.545
	0.636	0.727	0.818							
f =	35.4	36.1	36.0	33.0	30.6	31.7	34.0	34.0	35.7	39.5
	42.0	46.6	50.0							
r =	0.909	1.000	1.091	1.182	1.500	2.000	3.000	4.000	5.000	Inf.
f =	58.0	73	-	-	-	-	-	-	-	-
	74.0	77	72.0	72.0	74.0	71.0	73.5	73.0	73.0	72.0

It will be noted that these values are nearly constant up to one half diameter, followed by a slow rise to 58.0 at r = 0.909; but here the doubled frequency appears at 47. and remains nearly constant up to five diameters, and for a single cylinder.

Since the size of boiler installations will generally produce a spacing of the stacks of more than one diameter, with Moss Landing and Baltimore about 4.0 or a little more for example, it would appear that each stack would be "on its own" so far as wake and frequency are concerned. However, to revert to the previous discussion, it is unfortunate that no tests are available for a "quartering" wind such as caused the swaying of the Baltimore stacks. For such a case it seems most likely that interference will develop as described in Reference 2 of Separate 540. There remains to be determined how stacks "in echelon" behave, i.e. whether the interference of the wakes is similar to what is found here when the stacks are subjected to a "normal" wind.

F. B. FARQUHARSON,[13] M. ASCE.—This paper provides essential information regarding the wind-excited oscillation of a number of large steel stacks and opens up possibilities for much general discussion.

Under "Source of Vibrations" the authors have made reference to experimental work on suspension bridges. The statement regarding a velocity ratio of two between the first and second appearance of the same mode applies only to a very limited category of suspension bridge sections. This ratio was found only on plate-girder-stiffened sections where the ratio of depth of the girder to width of the section (d/b) was approximately 0.2. The same ratio was found to exist in the case of both torsional and vertical (or bending) modes and was observed on dynamic section models as well as on full models of the complete bridge.(5)

Two appearances of each vibration mode have been observed on other types of suspension bridge sections but never with a ratio of two between their critical velocities. It is also important to note that this multiple appearance of the same mode (under some circumstances there are more than two appearances) is assumed to indicate a condition of sub-harmonic resonance.

It does not necessarily follow that the later manifestations of a given mode must be catastrophic. Theoretical considerations have revealed the probable existence of other manifestations of both torsional and vertical vibration modes at still lower wind velocities. However, their development in the wind tunnel has been prevented due to inability to sufficiently reduce the structural damping inherent in the model suspension.

It appears that the exciting mechanism involved in suspension bridges of the girder-stiffened type with d/b = 0.22 has its origin in the discharge of vortices. For sections with d/b = 0.225 another type of excitation is superimposed on the vortex action.

The only similarity between the vibration of cylindrical stacks and sus-

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pension bridges like the original Tacoma Narrows Bridge lies in the fact that they are both vortex-excited. It would appear that the steel stack is harmonically excited by the vortex discharge and that a single appearance of the vibration mode will develop.

An excellent parallel exists between the cylindrical stack and the vibrating transmission line. In the course of an investigation of the high frequency vibration of power transmission lines (not to be confused with the galloping of such lines when ice coated) a series of tests was run on smooth cylinders of several diameters. In these tests, which were run on section models, one vertical mode only was isolated and it was quite evident that the excitation was harmonic with the exciting wind velocity being determined by the natural frequency of the model in its spring mounting. That is, at resonance which corresponds with maximum amplitude of vibration, the vortex frequency matches the frequency of vibration. As the wind velocity increases above the resonant value the vortex frequency increases in proportion to the velocity increase and eventually falls so far out of step with the natural frequency that the model comes to rest. As the wind velocity is reduced from its resonant value the model again comes to rest. In other words, the vortex discharge is able to excite motion only over a relatively small range of wind velocity either side of resonance.

In considering the aerodynamic forces acting on the stack it has been assumed that $C_D = 0.35$ and this appears to be a reasonable value since the field conditions involve operation in the vicinity of the critical value of R. However, the value of C_L, the alternating lift coefficient, which is given as $C_L = 0.66$ is at variance with the value computed by Landweber[16] as $C_L = 3.8\ C_D = 1.33$. It is assumed that the value used in the paper is taken from reference.[8]

It is a matter of great interest that the field observations on the vibration of a number of steel stacks has shown resonant vibration at values of Reynolds' number well beyond the range where many careful laboratory investigations have established a transition to turbulent flow. Figure A shows the variation of drag coefficient C_D and the Stroughal number $S = \frac{fd}{v}$ with Reynolds' number R as obtained on circular cylinders in two laboratories. Evidently regular vortex shedding did not occur beyond $R \approx$ 200,000 - 550,000. The following discussion may shed some light on this discrepancy.

Fage and Warsap[17] measured the drag of a 6.09-inch cylinder, when wires of various diameters were placed along generators at $\theta = \pm 65^{\circ}$ (Fig. B). The results of these tests are summarized in Fig. C where it will be noted that the shape of the drag curve suffered a regular change as the diameter of the wires was increased, with the largest wire producing the most marked change. When the angle θ was changed to $\pm 25^{\circ}$ the same wires had no appreciable effect.

Similar tests from the same source investigated the effect of uniform roughening developed on the surface of the cylinders by covering them with fine sandpaper of various coarseness. In Fig. D it will be seen that the drop in C_D occurs at a lower value of R, and becomes smaller, as the surface roughness is progressively increased. The range of velocities covered in these tests corresponds to Reynolds' numbers between 45,000 and 210,000 with the higher value approximating the critical (R = 6380 VL).

Many laboratory tests have also demonstrated that the critical range of Reynolds' number is also markedly reduced by increased turbulence in the wind stream.

Observations in the field on a number of stacks have revealed oscillations at values of R as high as 5,500,000 which is well above the critical values shown in Fig. A which are commonly accepted in the classical literature. (4), (18) It will be seen that two sources reporting in Fig. A place the critical value of R between 200,000 and 550,000 with the higher value developing in the wind tunnel with the lowest turbulence. These values are all determined on highly polished cylinders.

The results of tests by Fage and Warsap shown in Figs. C and D suggest that the coarse texture of the surface of the typical stack plus the presence on many of them of either a series of lapped and riveted joints or butt-welded seams, has provided a partial explanation of oscillations at high Reynolds' numbers observed in the field. In Fig. C it is evident that the largest wire used reduced C_D to 0.59 at V = 30 ft/sec (R = 97,000) where the bare cylinder showed the same value of C_D at V = 63 ft/sec (R = 204,000). This effect alone has resulted in a 47.5 per cent reduction in critical R.

Fig. D indicates that the effect of surface roughness was most pronounced with the 0.058 Garnet paper at C_D = 0.52 with a reduction of 47 per cent in R.

A measure of the effect of turbulence is found in Fig. A where the Göttingen tunnel is known to have the least turbulent flow and the critical value of R is found to be markedly lower for the more highly turbulent N.P.L. tunnel. The turbulence in both of these wind tunnels is of a much lower order than that experienced in natural wind; thus the effect of field turbulence could be expected to be somewhat greater.

Goldstein(18) has summarized this situation as follows: . . . the critical range of R, in which the drag coefficient experiences a large drop, can be changed progressively by ordered changes in the turbulence of the general stream, in the general goughness of the surface and by small excrescences . . . on the surface near the position of maximum negative pressure.

An examination of the data assembled from the six steel stacks which have oscillated under wind confirms the conclusion towards which this discussion has been directed. In the case of all but the Guam stack the evidence seems to point conclusively to a condition of harmonic resonance between the vortex frequency and the natural frequency of the fundamental mode of the stack.

The Guam report presents somewhat conflicting evidence. It seems possible that ovalling at 1.75 cps could have been of a sub-harmonic origin and might have coupled with a sub-harmonically excited bending mode. The frequency of the vortex shed by the undistorted stack would have been $f \approx 3.88$ cps for a Stroughal number of 0.19.

Under "Discussion of Data," a warning is sounded regarding the danger of reducing the critical velocity through added damping, the fear being expressed that a sub-harmonic excitation might occur at a higher wind velocity. The laboratory tests on circular cylinders mentioned above have indicated that only one appearance of the fundamental mode is possible. Of course there is always the possibility that the stack might vibrate in a higher mode; that is, with a different wave form. As the authors have mentioned for a stack of constant cross-section, the frequency in the 2nd mode is 6.25 times that of the 1st mode. Bearing in mind that the critical wind velocity varies directly with the natural frequency, it seems clear that for any but the most limber stacks the fundamental mode only need be considered.

In considering the "Design of Stacks," the suggestion that a sufficiently large diameter would yield a natural frequency above that of the vortex, is untenable. The increase in diameter would raise the natural frequency of the stack and increase the wind velocity at which the frequency of vortex dis-

charge would equal the frequency of the stack. The natural frequency can never exceed that of the vortices unless Reynolds' number is so high that turbulence inhibits the regularly discharged vortex pattern.

It was further suggested as a design procedure that stacks be designed for a natural frequency equivalent to the vortex frequency at 45 mph, and that the stack would be safe until a velocity of 80 mph was reached. This procedure simply involves a harmonic resonant condition in the vicinity of 45 mph which could be disastrous under low damping conditions. At 80 mph the Moss Landing stack would operate at R = (6380)(117)(11.3) = 8,450,000, which, even allowing for turbulence and roughness corrections, should place the stack above the critical value of R where vortices are not regularly shed.

In line with "Further Investigations," it is felt that items 1 to 3 need no further study since the wind force on a fixed member controls if means can be found for eliminating vibration. The present discussion is submitted as an answer to item 4. The damping characteristics mentioned in item 5 are important since this feature represents one element in the control of the critical velocity. The elimination of vortex shedding (item 6) is probably a practical impossibility due to the fact that the wind may blow from all points of the compass.

One might suggest the following investigations:

1. The design of a practical stiffening ring to eliminate ovalling.
2. The mounting of vertical vanes in the manner of the wires in Fig. B, placed 45^{o}, 51.5^{o} or 60^{o} apart completely around the stack and with such a depth as to stay within the boundary layer. It may be assumed that such a device might serve to reduce the drag much as shown in Fig. C and thus also the strength of the vorticity. Wind tunnel experimentation would develop as optimum depth and spacing of these vanes.
3. Wind tunnel studies on a special type of flexible model to determine the amount of taper which might effectively eliminate oscillation. This device if successful would reduce the design wind loads to those arising from the alternating lateral force which is at least twice as great as the drag force.

ADDITIONAL REFERENCES

16. Landweber, L., "Flow about a Pair of Adjacent, Parallel Cylinders Normal to a Stream," Taylor Model Basin Memorandum, January, 1945.
17. Fage and Warsap, A. R. C. Reports and Memoranda, (British) No. 1238 (1930).
18. Goldstein, S., "Modern Developments in Fluid Mechanics," Vol. II, Oxford University Press.

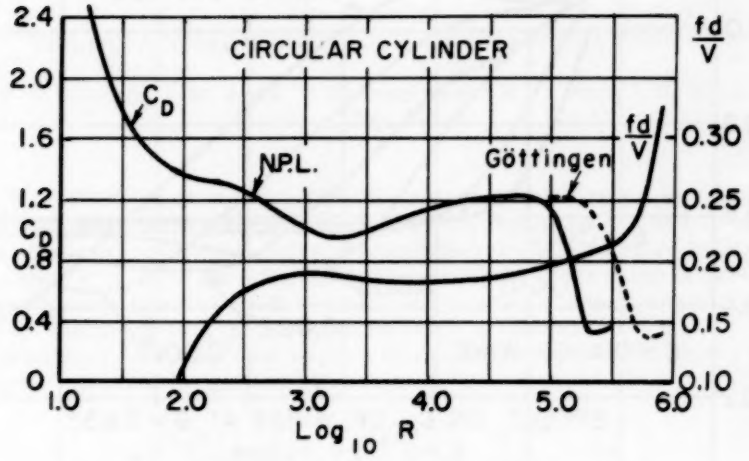
CIRCULAR CYLINDER
C_D
N.P.L.
Göttingen
$\frac{fd}{V}$
C_D
2.4
2.0
1.6
1.2
0.8
0.4
0
0.30
0.25
0.20
0.15
0.10
1.0
2.0
3.0
4.0
5.0
6.0
Log 10 R

Fig. A

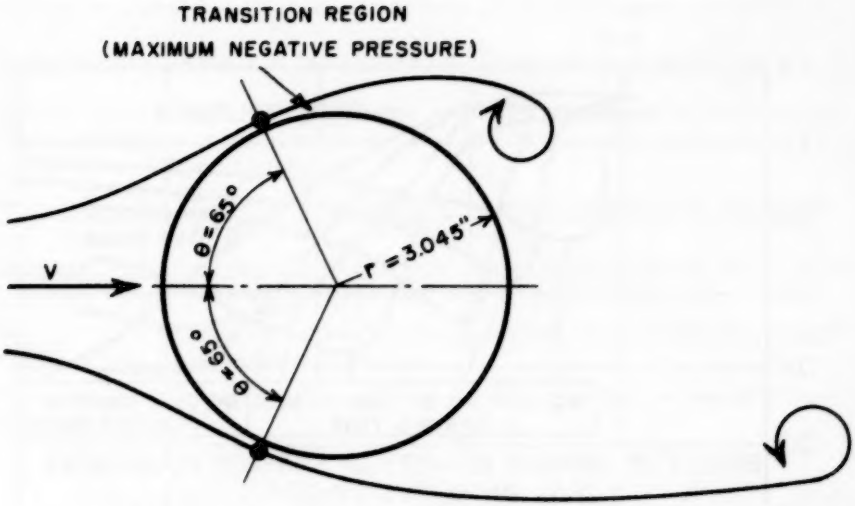
TRANSITION REGION
(MAXIMUM NEGATIVE PRESSURE)
V
θ=65°
θ=65°
r = 3.045"

Fig. B

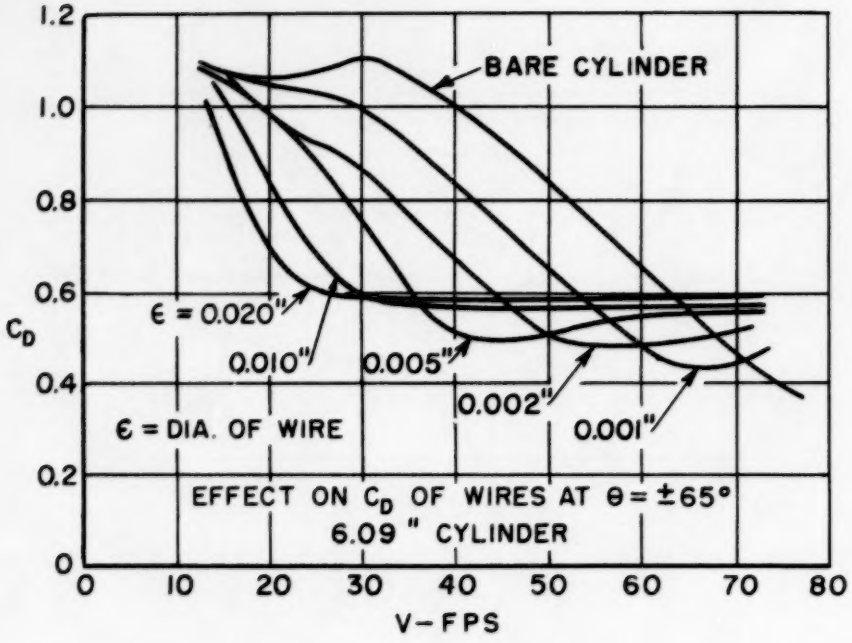
1.2
1.0
0.8
0.6
0.4
0.2
0
C_D
BARE CYLINDER
ϵ = 0.020"
0.010"
0.005"
0.002"
0.001"
ϵ = DIA. OF WIRE
EFFECT ON C_D OF WIRES AT $\theta = \pm 65°$
6.09 " CYLINDER
0 10 20 30 40 50 60 70 80
V-FPS

Fig. C

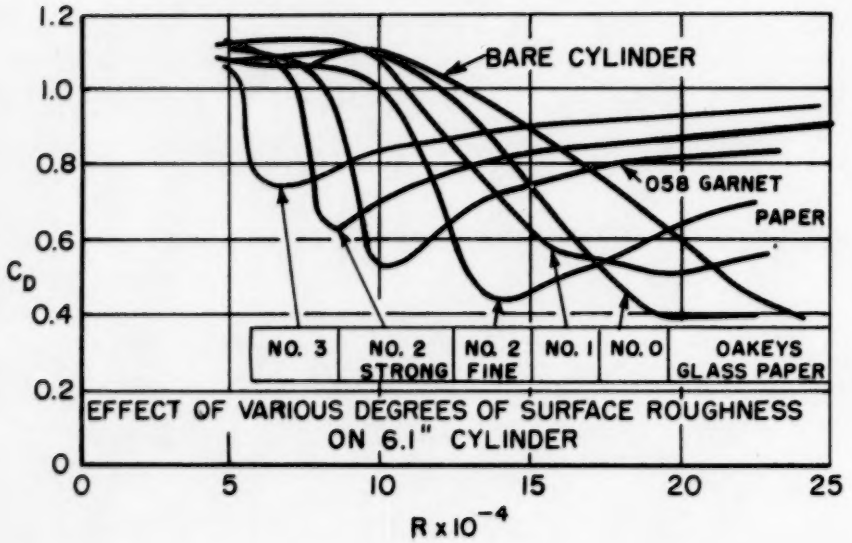
1.2
1.0
0.8
0.6
0.4
0.2
0
C_D
BARE CYLINDER
058 GARNET PAPER
NO. 3
NO. 2 STRONG
NO. 2 FINE
NO. 1
NO. 0
OAKEYS GLASS PAPER
EFFECT OF VARIOUS DEGREES OF SURFACE ROUGHNESS
ON 6.1" CYLINDER
0 5 10 15 20 25
R x 10^{-4}

Fig. D

Discussion of
"THE OCTAGONAL GIRDER FOUR COLUMN SPACE FRAME"

By P. C. Disario, A.M. ASCE, J. S. Podolan and N. A. Weil, J.M. ASCE
(Proc. Sep. 542)

A. F. DIWAN,[1] A.M. ASCE.—The authors' paper deals with a special type of space frames, namely the octagonal girder rigidly supported on four columns. The work involved is indeed remarkable, although its value is limited to the narrow scope of the octagonal girder under the given cases of loading. The coefficients obtained for the bending and twisting moments are very useful to all concerned with the design of this type of frames.

The authors have used Castiliagno's theorem in computing the angles of rotation θ, as given by equation (5) below;

$$\theta_A^T = \frac{\partial U}{\partial T'} \left[\Sigma \int \left(\frac{M^2}{2EI} + \frac{T^2}{2GJ} \right) dx \right]$$

$$= \frac{1}{EI} \left[\Sigma \int \left(M \frac{\partial M}{\partial T'} + KT \frac{\partial T}{\partial T'} \right) dx \right] \quad \ldots (5)$$

The writer however finds it much easier to use the method of "Virtual Work"; which he regards as a refined form of the Castiliagno equations.

Referring to equation 5, it can be seen that the terms $\frac{\partial M}{\partial T'}$ and $\frac{\partial T}{\partial T'}$ are in fact the bending and twisting moments respectively produced by unit torques T' = 1.0 acting at ponts A and C.

Using the following notations:

M_m , T_m — for bending and twisting moments respectively, due to unit moments M = 1 at B and D.,

M_t , T_t — for bending and twisting moments respectively, due to unit torques T = 1 at A and C.,

M_o , T_o — for bending and twisting moments respectively, due to any given case of loading.,

then, the angles of rotation θ for this given case of loading as obtained from equation 5 will be:

$$\theta_A^T = \frac{1}{EI} \left[\Sigma \int \left(M_o M_t + K T_o T_t \right) dx \right] \cdots\cdots (5\text{-b})$$

$$\theta_B^M = \frac{1}{EI} \left[\Sigma \int \left(M_o M_m + K T_o T_m \right) dx \right] \cdots\cdots (5\text{-c})$$

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in which $k = \frac{EI}{GJ}$

These are the equations of "Virtual work." The problem thus reduces to integrating two moment diagrams for each member. The first of these diagrams corresponds to the given load system producing the rotation θ, and the second to the case of loading M = 1.0 or T = 1.0. The second of these diagrams being linear, the integration for a prismatic member is simply obtained as $A_o M^*$ or $A_o T^*$ where:

A_o = Elastic area under the first diagram (M_o or T_o)

M^*, T^* = ordinate in the second diagram (M_t, M_m, T_t, T_m) under the c.g. of the first diagram.

Figure (1) shows three moment diagrams, the first two of which may correspond to the case M_o, and the third to the case M_m say. When both diagrams M_o and M_m (or T_o & T_m . . . . etc.) are linear, as that in the cases b & c fig. (1) say, the integration will become:

$$I = \frac{\bar{L}}{3}\left[2(m_1 m_3 + m_2 m_4) + (m_1 m_4 + m_2 m_3)\right] \quad (1)$$

In the special case when the diagrams b and c are similar, i.e. when integrating diagram c by itself, the result will be,

$$I_c = \frac{\bar{L}}{3}\left(m_1^2 + m_2^2 + m_1 m_2\right) \quad \ldots \quad (2)$$

where:

$\bar{L} = \frac{L}{EI}$ for bending diagrams, and $\frac{L}{GJ}$ for twisting moment diagrams.

Now consider equation (23) of the authors, that is:

$$\theta_B^M = \frac{a}{EI}\left(\int_3 \bar{M} + \int_1 \bar{T}\right) \quad \ldots . (23)$$

In this equation, it can be seen that $\int_3$ will be the rotation $\frac{EI}{a} \cdot \theta_B^M$ due to unit bending moments M = 1.0 acting at B and D, so that:

$$\int_3 = \frac{1}{a}\left[\Sigma \int \left(M_m^2 + K T_m^2\right) dx\right] \quad \ldots . \quad (3)$$

The integration is being carried over two quadrants of the girder. Figure 2, shows the bending and twisting moment diagrams M_m and T_m for one quadrant. The redundant bending moment in this case is: (see eqn. 21)

$$M'' = C = \frac{0.3536\,k + 0.3018}{k+1}$$

Using eqn. (2), the integration of eqn. (3) over two quadrants will give:

$$\xi_3 = \frac{0.0858\,k^2 + 0.0638\,k + 0.541}{k+1} \quad \ldots (4)$$

This value for ξ_3 differs from that given by the authors, equation (26). The other two functions however, ξ_1 and ξ_2 obtained by the authors are correct. This has influenced the values for all the moment coefficients γ and ζ given by the authors. These coefficients will have now to be revised.

The solution of the problem using the classical methods of Castiliagno or virtual work as given by the authors is too complex, although the derivation of the authors formulae is indeed excellent. The writer has, however, developed a simple method for the analysis of statically indeterminate frames loaded normal to their plane using the concept of "elastic poles." The method is easily applied in the analysis of space frames, such as that dealt with in the authors' paper. This is the subject of a paper which the writer hopes will be ready for presentation soon. Applying his method to the problem of the octagonal girder, the writer got the following functions for the moment coefficients;

$$\gamma_1 = \left[n(k+1) + 1.81\,k^2 + 5.587\,k + 0.466 \right] \div 3E \quad (5)$$

$$\gamma_2 = \frac{1}{3E}\left[n(1.914\,k + 0.707) + 1.81\,k^2 + 5.56\,k + 0.466 \right] \quad (6)$$

$$\gamma_3 = \frac{nA}{4B} \quad \ldots\ldots \quad (7)$$

$$\gamma_4 = \frac{nC}{4B} \quad \ldots\ldots \quad (8)$$

$$\gamma_5 = \frac{2.9\,nE^2}{4B} \quad \ldots\ldots \quad (9)$$

$$\zeta_1 = \frac{-1}{E}\left[0.096 + 0.079\,k + 0.021\,k^2 \right] \ldots (10)$$

$$\zeta_2 = \frac{1}{E}\left[0.19 + 0.883\,k + 0.279\,k^2 + n(0.212 + 0.238\,k) \right] \quad (11)$$

in which:

$$A = 5.8\,n^2(k+1)^2 + n(22.7\,k^3 + 95\,k^2 + 85\,k + 12) + (21\,k^4 + 135\,k^3 + 244\,k^2 + 83\,k + 5.5) \quad (12)$$

$$B = 23\,n^3(k+1) + n^2(80\,k^3 + 324\,k^2 + 340\,k + 76) + n(65\,k^4 + 466\,k^3 + 985\,k^2 + 522\,k + 66) + (6\,k^5 + 85\,k^4 + 366\,k^3 + 551\,k^2 + 167\,k + 7) \quad (13)$$

$$C = n^2(k+1)(11\,k + 4.1) + n(30.7\,k^3 + 117\,k^2 + 85\,k + 10) + (21\,k^4 + 135\,k^3 + 244\,k^2 + 83\,k + 5.5) \quad (14)$$

$$E = 3 + 10\,k + 3k^2 + 2.667\,n(k+1) \quad (15)$$

For the given dimensions of girder and columns, the writer got the following numerical values for the moment coefficients.

$\gamma_1 = 0.144$	(0.150)
$\gamma_2 = 0.173$	(0.175)
$\gamma_3 = 0.062$	(0.0672)
$\gamma_4 = 0.073$	(0.075)
$\gamma_5 = 0.1075$	(0.1086)

$\zeta_1 = 0.00445$	(0.00467)
$\zeta_2 = 0.0855$	(0.086)
$\zeta_3 = 0.0085$	(0.0090)
$\zeta_4 = 0.083$	(0.082)
$\zeta_5 = 0.0056$	(0.0057)

The values obtained by the authors are shown in brackets.

Throughout the computations, the writer has used an ordinary 12" slide rule, and no simultaneous equations were necessary in the solution.

The great value of the authors' paper is in the closed form coefficients obtained for the moments. But, as the authors themselves admit, the expressions given for these moment coefficients are extremely complex, and require a great deal of computations to get the numerical values for any given n & k values. This overshadows the real value of the paper, and it became necessary for the authors to seek a modified simple analysis. To make these moment coefficients easily applicable, they should be given in a curve form, a thing which appeals to all engineers. The writer has put the coefficients of equations (5) to (11) in a curve form shown in figures (3) to (9).

These curves cover the ranges k = 0 to k = 3 and n = 0 to n = 10. The authors may indeed extend these curves, with more accuracy, to a wider range.

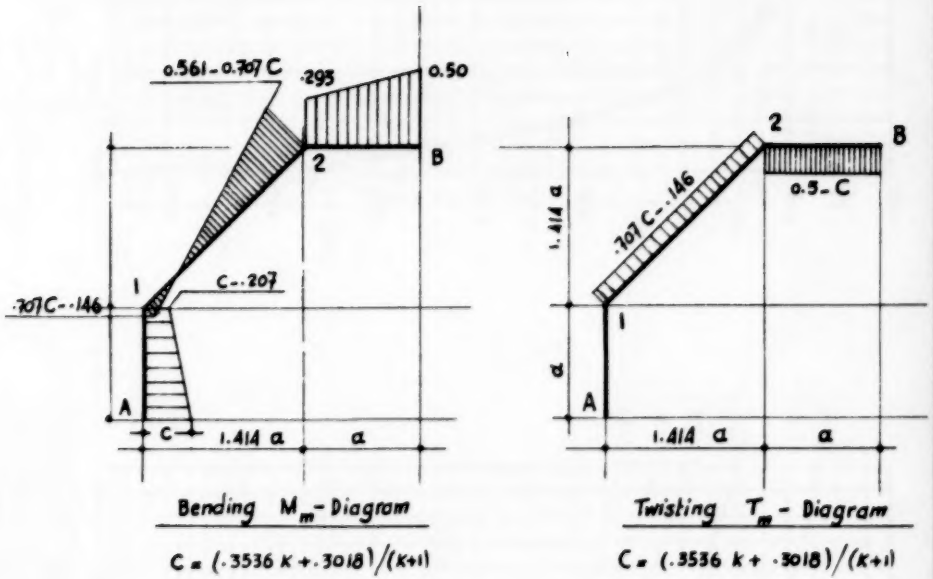
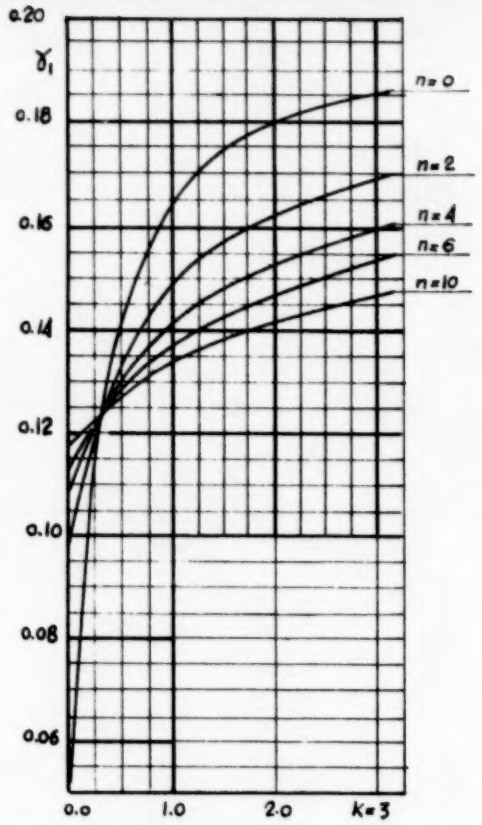
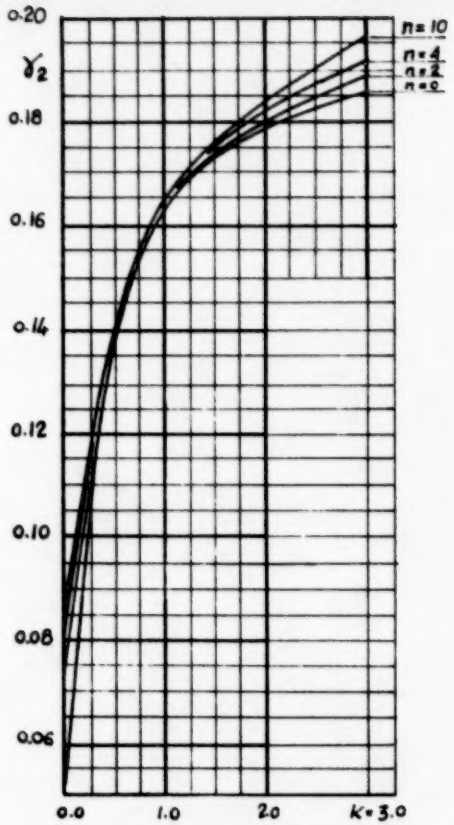

0.561 - 0.707 C
.293
0.50
2
B
1
C - .207
.707 C - .146
A
C
1.414 a
a
Bending M_m - Diagram
C = (.3536 K + .3018)/(K+1)
2
B
0.5 - C
1.414 a
.707 C - .146
1
a
A
1.414 a
a
Twisting T_m - Diagram
C = (.3536 k + .3018)/(K+1)

FIG. (2)



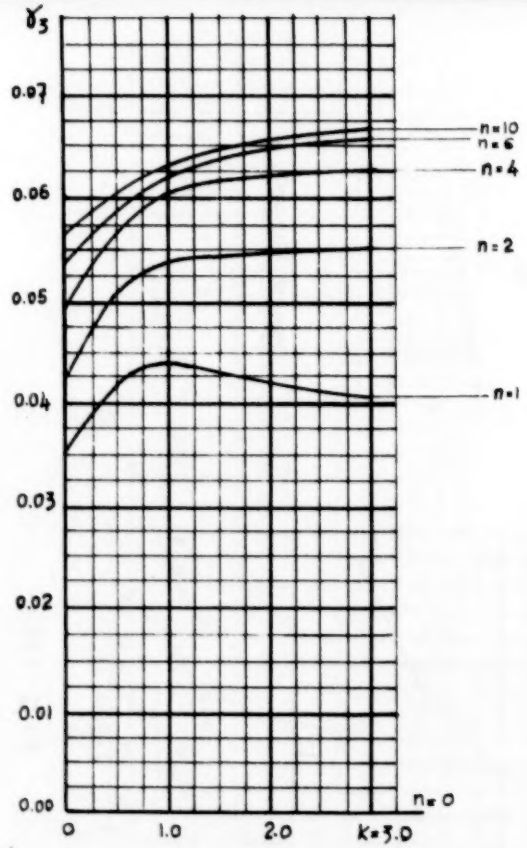
$\gamma_1 = M_B / Ph$ (Table 3)

FIG. (3)



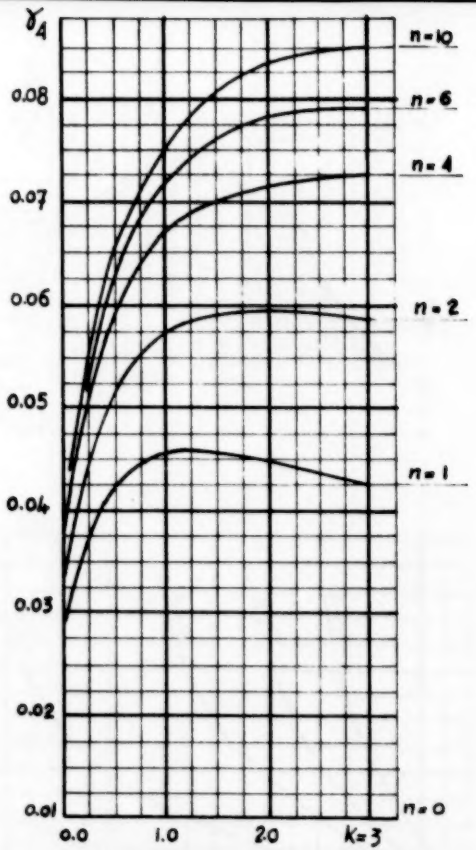
$\gamma_2 = -M_A / Ph$ (Table 3)

FIG. (4)



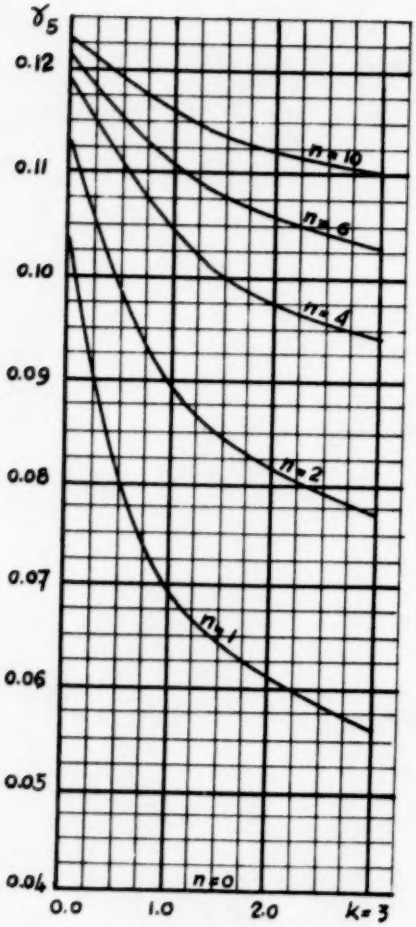
$-\gamma_3 = M_B / Ph$
(Table 4)

FIG (5)



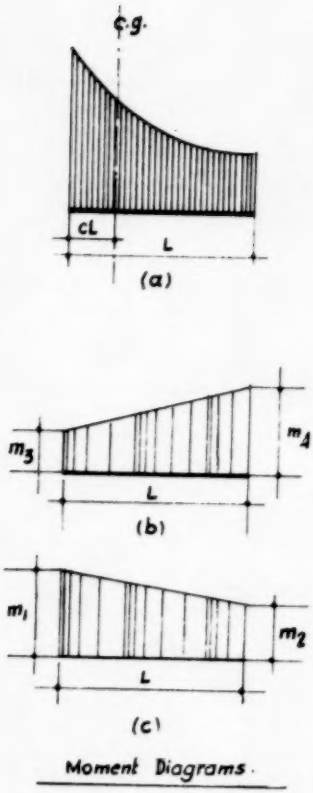
$\gamma_4 = -M_A / Ph$
(Table 4)

FIG (6)



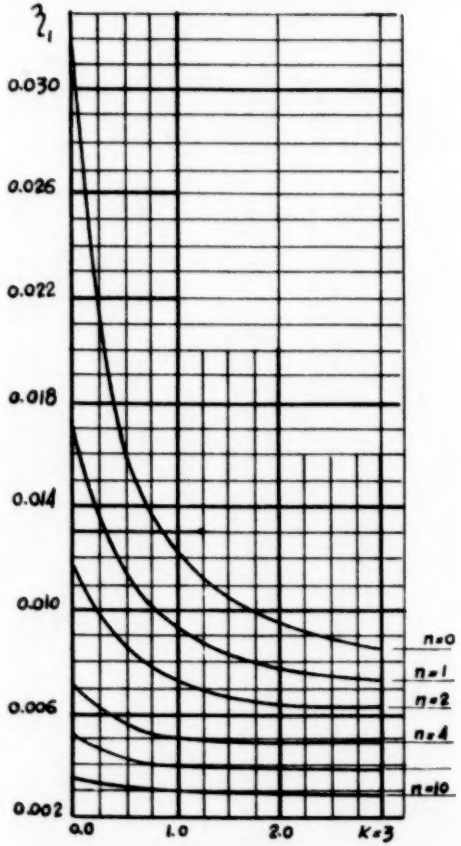
γ_5 in $T_A = (\gamma_3 - \gamma_5)$ Ph.
(Table 4)

FIG. (7)



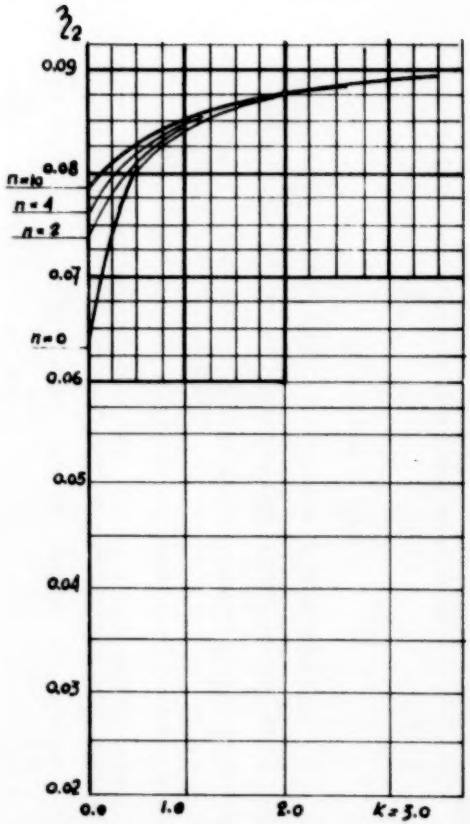
Moment Diagrams.

FIG. (1)



$\zeta_1 = -M_B/M_o$ (Table 5)

FIG. (8)



$\zeta_2 = -M_A/M_o$ (Table 5)

FIG (9)

PROCEEDINGS-SEPARATES

The technical papers published in the past year are presented below. Technical-division sponsorship is indicated by an abbreviation at the end of each Separate Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways (WW) divisions. For titles and order coupons, refer to the appropriate issue of "Civil Engineering" or write for a cumulative price list.

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APRIL: 428(HY)[c], 429(EM)[c], 430(ST), 431(HY), 432(HY), 433(HY), 434(ST).

MAY: 435(SM), 436(CP)[c], 437(HY)[c], 438(HY), 439(HY), 440(ST), 441(ST), 442(SA), 443(SA).

JUNE: 444(SM)[e], 445(SM)[e], 446(ST)[e], 447(ST)[e], 448(ST)[e], 449(ST)[e], 450(ST)[e], 451(ST)[e], 452(SA)[e], 453(SA)[e], 454(SA)[e], 455(SA)[e], 456(SM)[e].

JULY: 457(AT), 458(AT), 459(AT)[c], 460(IR), 461(IR), 462(IR), 463(IR)[c], 464(PO), 465(PO)[c].

AUGUST: 466(HY), 467(HY), 468(ST), 469(ST), 470(ST), 471(SA), 472(SA), 473(SA), 474(SA), 475(SM), 476(SM), 477(SM), 478(SM)[c], 479(HY)[c], 480(ST)[c], 481(SA)[c], 482(HY), 483(HY).

SEPTEMBER: 484(ST), 485(ST), 486(ST), 487(CP)[c], 488(ST)[c], 489(HY), 490(HY), 491(HY)[c], 492(SA), 493(SA), 494(SA), 495(SA), 496(SA), 497(SA), 498(SA), 499(HW), 500(HW), 501(HW)[c], 502(WW), 503(WW), 504(WW)[c], 505(CO), 506(CO)[c], 507(CP), 508(CP), 509(CP), 510(CP), 511(CP).

OCTOBER: 512(SM), 513(SM), 514(SM), 515(SM), 516(SM), 517(PO), 518(SM)[c], 519(IR), 520(IR), 521(IR), 522(IR)[c], 523(AT)[c], 524(SU), 525(SU)[c], 526(EM), 527(EM), 528(EM), 529(EM), 530(EM)[c], 531(EM), 532(EM)[c], 533(PO).

NOVEMBER: 534(HY), 535(HY), 536(HY), 537(HY), 538(HY)[c], 539(ST), 540(ST), 541(ST), 542(ST), 543(ST), 544(ST), 545(SA), 546(SA), 547(SA), 548(SM), 549(SM), 550(SM), 551(SM), 552(SA), 553(SM)[c], 554(SA), 555(SA), 556(SA), 557(SA).

DECEMBER: 558(ST), 559(ST), 560(ST), 561(ST), 562(ST), 563(ST)[c], 564(HY), 565(HY), 566(HY), 567(HY), 568(HY)[c], 569(SM), 570(SM), 571(SM), 572(SM)[c], 573(SM)[c], 574(SU), 575(SU), 576(SU), 577(SU), 578(HY), 579(ST), 580(SU), 581(SU), 582(Index).

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JANUARY: 583(ST), 584(ST), 585(ST), 586(ST), 587(ST), 588(ST), 589(ST)[c], 590(SA), 591(SA), 592(SA), 593(SA), 594(SA), 595(SA)[c], 596(HW), 597(HW), 598(HW)[c], 599(CP), 600(CP), 601(CP), 602(CP), 603(CP), 604(EM), 605(EM), 606(EM)[c], 607(EM).

FEBRUARY: 608(WW), 609(WW), 610(WW), 611(WW), 612(WW), 613(WW), 614(WW), 615(WW), 616(WW), 617(IR), 618(IR), 619(IR), 620(IR), 621(IR)[c], 622(IR), 623(IR), 624(HY)[c], 625(HY), 626(HY), 627(HY), 628(HY), 629(HY), 630(HY), 631(HY), 632(CO), 633(CO).

MARCH: 634(PO), 635(PO), 636(PO), 637(PO), 638(PO), 639(PO), 640(PO), 641(PO)[c], 642(SA), 643(SA), 644(SA), 645(SA), 646(SA), 647(SA)[c], 648(ST), 649(ST), 650(ST), 651(ST), 652(ST), 653(ST), 654(ST)[c], 655(SA), 656(SM)[c], 657(SM)[c], 658(SM)[c].

APRIL: 659(ST), 660(ST), 661(ST)[c], 662(ST), 663(ST), 664(ST)[c], 665(HY)[c], 666(HY), 667(HY), 668(HY), 669(HY), 670(EM), 671(EM), 672(EM), 673(EM), 674(EM), 675(EM), 676(EM), 677(EM), 678(HY).

c. Discussion of several papers, grouped by Divisions.
e. Presented at the Atlantic City (N.J.) Convention in June, 1954.

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